

1, Prove that

$$(p \rightarrow r) \vee (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r$$

2. Prove that

$$\neg(p \leftrightarrow q) \leftrightarrow (p \leftrightarrow \neg q)$$

3. Show that there are 16 different propositional formulas in p and q . But \neg, \wedge, \vee are the only ones one needs to express all others. For example $p \rightarrow q \leftrightarrow \neg p \vee q$

In general any propositional formula $P(p_1, \dots, p_n)$ is equivalent to a disjunction of conjunctions in p_i and $\neg p_j$. Similarly, any $P(p_1, \dots, p_n)$ is equivalent to a conjunction of disjunctions in p_i and $\neg p_j$.